

# Nucleus from String Theory

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In generic holographic QCD, we find that baryons are bound to form a nucleus, and that its radius obeys the empirically-known mass number ( $A$ ) dependence  $r \propto A^{1/3}$  for large  $A$ . Our result is robust, since we use only a generic property of D-brane actions in string theory. We also show that nucleons are bound completely in a finite volume. Furthermore, employing a concrete holographic model (derived by Hashimoto, Iizuka, and Yi, describing a multi-baryon system in the Sakai-Sugimoto model), the nuclear radius is evaluated as  $\mathcal{O}(1) \times A^{1/3}$  [fm], which is consistent with experiments.

To describe atomic nuclei directly by strongly coupled quark dynamics, QCD, is a long-standing problem in nuclear physics and particle physics. It was only recent that lattice QCD simulations reproduce qualitatively the nuclear forces. Recent progress in solving strongly coupled gauge theories with a new mathematical tool of superstring theory, the AdS/CFT correspondence, has been proven to be truly powerful in application to QCD (called holographic QCD).

In this letter, we show by quite a generic argument in superstring theory and the AdS/CFT correspondence that non-supersymmetric QCD-like theories in the large  $N_c$  limit host nuclei, multi-baryon bound states. Furthermore, we can show also that the resultant nuclei have the important nuclear property in the real world: Finiteness of the nuclear size, and its mass-number dependence. That is, the radius of the holographically realized nuclei is shown to be proportional to  $A^{1/3}$  where  $A$  is the mass number (the baryon number) of the nucleus.

In deriving these, we do not rely on any specific model of holographic QCD. What we use is only the following two known facts: (i) Baryons are D-branes in any gravity dual of QCD-like gauge theories [2, 3], (ii) D-brane effective actions are a dimensionally reduced Yang-Mills (YM) theory [4]. From these two, the formation of the nuclei and the mass-number dependence of the nuclear size follow. Therefore our finding is quite robust and universal for any holographic description of non-supersymmetric QCD-like gauge theories, at the large  $N_c$  and at the strong coupling.

Our derivation is divided into two steps.

1. The system with a large number of baryons in generic holographic QCD is described effectively by a simple bosonic matrix quantum mechanics. It is a pure YM action dimensionally reduced to 1 dimension [26],

$$S = c \int dt \operatorname{tr}_A \left[ \frac{1}{2} (D_t X^I)^2 + \frac{g^2}{4} [X^I, X^J]^2 \right]. \quad (1)$$

where  $I = 1, \dots, D$ . The eigenvalues of the  $A \times A$  matrix  $X^i$  ( $i = 1, 2, 3$ ) are location of the  $A$  baryons in our space, and  $X^{\hat{i}}$  ( $\hat{i} = 4, \dots, D$ ) is for holographic directions.

2. The system allows a non-perturbative vacuum at which the eigenvalues of  $X^i$  form a ball-like distribution, which is nothing but a nucleus. The size shows the mass-number dependence  $A^{1/3}$ .

In the following, we shall show 1 and 2 in turn. Finally we present the explicit form of the nuclear density distribution (10). Together with the explicit matrix model [1] where input parameters are only the  $\rho$  meson mass and the  $NN\pi$  coupling, we obtain the nuclear radius  $R \sim \mathcal{O}(1) \times A^{1/3}$  [fm], which is consistent with the standard experimental observation.

**Multi-baryons described by matrices.** — First, let us show that the system of  $A$  baryons in generic holographic QCD is dictated by the matrix quantum mechanics (1). This can be provided by known facts (i) and (ii) which we explain below.

(i) *Baryons are D-branes in any gravity dual of QCD-like gauge theories*, basically because baryons in large  $N_c$  gauge theories are heavy as their mass diverges as  $\mathcal{O}(N_c)$  while D-branes are solitons of string theory and thus heavy. In fact, from the point of view of baryon charges, it is known that, in the gravity dual side of strongly coupled gauge theories, D-branes wrapping compact cycles can be identified as baryons [2, 3]. This is natural since in large  $N_c$  QCD baryons appear as solitons of meson effective field theory, as in the famous Skyrme model.

To show the universality of the statement, let us illustrate this fact with a specific and concrete dual gravity background provided by Witten [6]. The spacetime ends smoothly at an IR end capped with non-contractable  $S^4$ . A D-brane corresponding to a baryon is the one wrapping it (note that it is orthogonal to our spacetime so the D-brane looks as a point particle in our space). As the  $S^4$  is supported by  $N_c$  units of a Ramond-Ramond (RR) flux in the supergravity solution of the geometry, the wrapped D-brane, called a baryon vertex, has to attach the ends of  $N_c$  fundamental strings, thus attain a single baryon charge ( $= N_c$  quark charge) of the gauge theory [2]. This situation is universal in holographic QCD, by the following two reasons. First, it is expected that, in any gravity dual of confining gauge theories, the geometry pos-

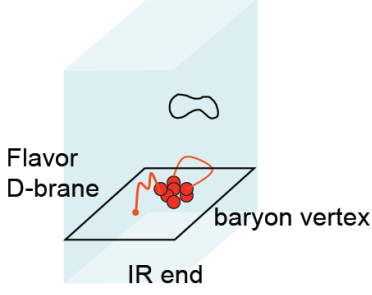


FIG. 1: Gravity dual of QCD-like confining gauge theories with  $A$  baryons. The horizontal directions are our 3-dimensional space, and the vertical direction is the holographic direction. This 10-dimensional spacetime is curved, and the bottom is capped by a smooth compact manifold, which is the IR end. The baryon vertices ( $A$  red balls) sit on the  $N_f$  flavor D-branes. The degrees of freedom of the baryons are open strings attached to the baryon vertices.

sesses a smooth IR end accompanying a compact manifold, known as the name of “confining geometry” [27]. This end is necessary to provide a finite QCD string tension. Second, the compact manifold always carries the  $N_c$  RR flux because the geometry is a gravity dual of  $SU(N_c)$  gauge theories realized by  $N_c$  D-branes.

Any nucleus is labeled by spins and isospins, so we need to assign flavor (isospin) charges on the baryon in the AdS/CFT. Here we review the standard holographic description of the flavor symmetry. The quark sector of QCD-like gauge theories is realized in the AdS/CFT correspondence by introducing flavor D-branes [8]. Typically we need  $N_f$  D-branes which extend also along our spacetime directions, where  $N_f$  is the number of quark flavors. The  $N_c$  fundamental string emanating from the baryon vertex end on these flavor branes. The excitation of these strings represent the flavor charges of the baryons. This is nicely realized in Sakai-Sugimoto model of holographic QCD [9]. Quantum excitations of the strings give rise to the spins and isospins [1] [28]. In Fig. 1, we show a typical brane configuration of gravity dual of confining gauge theories with  $A$  baryons.

(ii) *Effective theory on the  $A$  baryon vertices for large  $A$  is given by the matrix quantum mechanics (1).* Fundamental degrees of freedom on any collection of D-branes are strings connecting the D-branes. In our case, we have  $A$  baryon vertices and  $N_f$  flavor D-branes, thus we have  $A \times A$  matrix  $X^I$  from strings among the  $A$  D-branes, in addition to  $A \times N_f$  matrix  $w$  coming from strings connecting the baryon vertex end on these flavor branes. These strings are shown in Fig. 1. The effective theory is a  $U(A)$  gauge theory, where  $X^I$  is in the adjoint representation while  $w$  is in the fundamental representation. The index  $I$  runs for our spatial directions  $i = 1, 2, 3$ , and the holographic directions  $\hat{i} = 4, \dots, D$ .

Our interest is a large  $A$ , that is, a heavy nucleus, as  $A$  is the mass number. At the leading order in the large  $A$  expansion, the fundamental representation field

$w$  drops off. It is known in string theory that the  $X^I$  part has a universal effective action which is a dimensionally reduced YM action, (1).

We ignore the gauge field components along the compact manifold ( $S^4$  in the previous example), since those directions are irrelevant to QCD [29]. The time component gauge field  $A_t$  still remains, but it is not dynamical and just ensures the local gauge invariance [30]. Fermionic superpartners are heavy due to the supersymmetry breaking and assumed not effective.

An explicit action can be derived once we fix the species of the branes in holographic QCD. The nuclear matrix model of [1] [31] was derived in the Witten’s geometry with the flavor D8-branes of the Sakai-Sugimoto model,

$$S = \frac{\lambda N_c M_{\text{KK}}}{3^3 \pi} \int dt \text{tr}_A \left( \frac{1}{2} (D_t X^I)^2 + \frac{\lambda^2 M_{\text{KK}}^4}{3^6 \pi^2} [X^I, X^J]^2 - \frac{1}{3} M_{\text{KK}}^2 ((X^4)^2 + (X^5)^2) \right) + \text{sub-leading in } 1/A. \quad (2)$$

Here  $I$  runs from 1 to  $D = 5$ ,  $\lambda \equiv N_c g_{\text{QCD}}^2$  is the QCD ’tHooft coupling, and  $M_{\text{KK}}$  is a dynamical scale (roughly corresponding to the QCD scale). With a rescaling  $Y^I \equiv X^I (\lambda N_c M_{\text{KK}} / 3^3 \pi)^{1/2}$ , the leading order is written in a canonical expression,

$$S = \int dt \text{tr}_A \left( \frac{1}{2} (D_t Y^I)^2 - \frac{m_Y^2}{2} (Y^{\hat{i}})^2 + \frac{g_0^2}{4} [Y^I, Y^J]^2 \right), \quad (3)$$

where the matrix model coupling squared is  $g_0^2 \equiv 2^2 \lambda M_{\text{KK}}^3 / (3^3 \pi N_c)$ . In comparison to (1) here there is a mass term with  $m_Y^2 = (2/3) M_{\text{KK}}^2$ , but it is qualitatively irrelevant in our analysis, see Footnote [32]. The mass term is only along the holographic direction  $X^{\hat{i}}$ , which means that the baryon vertex is stable at the IR end of the geometry  $X^{\hat{i}} = 0$ .

**Formation of nucleus and nuclear size.** — We shall show that indeed the eigenvalues of the matrix  $X^I$  are bound to each other, which directly means a formation of a nucleus, since the eigenvalues are the location of the  $A$  baryons.

As the action (1) is a dimensional reduction of a YM theory, we can apply the generic argument by Lüscher [17] on spectra of YM theory on torii. Lüscher proved a theorem that the ground state of the theory (1) has eigenvalues bound to a finite region, and is invariant under the  $U(A)$  rotation and the spatial rotation. Even though there exists a flat direction of the potential at which all  $X^I$  are diagonal, the eigenvalues are bound because the flat direction is narrow for large values of  $X^I$  and the quantum dynamics suppresses percolation. So, *this theorem ensures a formation of a spherical nucleus in generic holographic QCD.*

Next, we show the nuclear radius  $\propto A^{1/3}$ . It follows simply from a dimensional analysis of the action (1). The

nuclear size is given by the distribution of the entries of  $X^i$  ( $i = 1, 2, 3$ ), so we can define the mean square radius as

$$r_{\text{mean}}^2 \equiv \frac{1}{A} \sum_{i=1}^3 \langle \text{tr}_A [X^i X^i] \rangle. \quad (4)$$

The expectation value is taken with the ground state of the matrix quantum mechanics. The normalization  $1/A$  is understood from the case of diagonal  $X$  with which individual location of the baryons makes sense. Using the properly normalized action as in (3), we can estimate

$$\frac{1}{A^2} \sum_{i=1}^3 \langle \text{tr}_A [Y^i Y^i] \rangle = c_0 \lambda_A^{-1/3} + O(1/A), \quad (5)$$

at large  $A$ , where  $\lambda_A \equiv A g_0^2$  is a 'tHooft coupling of the quantum mechanics and  $c_0$  is an undetermined  $A$ -independent dimensionless constant. We have used a standard 'tHooft expansion [18]. The  $\lambda_A$  dependence was determined from the mass dimension:  $[\lambda_A] = 3$  and  $[Y^i] = -1/2$ . ( $\lambda_A$  is the unique dimension-ful parameter in the action.) Since the rescaling from  $X$  to  $Y$  is  $A$ -independent, we obtain  $\sqrt{r_{\text{mean}}^2} \propto A^{1/3}$ . *Therefore we conclude that in generic holographic QCD a nucleus forms as a bound state of  $A$  baryons at large  $A$ , and exhibits the correct  $A$  dependence of the nuclear size.*

**Nuclear density distribution.** — A certain approximation of the matrix quantum mechanics (1) enables us to compute more detailed information of the bound state: nuclear density distribution. Here we shall show that the nuclear density computed by the matrix quantum mechanics (1) vanishes outside a certain radius, which is the finiteness of nuclei.

To this end, we employ a RR density formula [19] in a  $D$  dimensional spacetime, at the leading order in  $1/A$ , developed in the context of Matrix theory [20] in superstring theory:

$$\rho(x) = \frac{1}{(2\pi)^D} \int d^D k e^{-ik \cdot x} \langle \text{tr}_A \exp[ik \cdot X] \rangle. \quad (6)$$

Since our baryon vertices are RR-charged D-branes, (6) is equivalent to the baryon charge distribution. We evaluate the expectation value at zero temperature [32].

To evaluate (6) with the model (1), we need a help of a large  $D$  expansion of matrix models [21–23], where  $D$  counts the number of the matrices  $X^I$  ( $I = 1, \dots, D$ ). We fix  $\tilde{\lambda}_A \equiv g_0^2 A D$  to be finite, and take the large  $D$  and large  $A$  limits. The large  $D$  limit is known to be a good approximation as discussed in [22] even for small  $D$  ( $\geq 2$ ) qualitatively. According to [22], at the leading order of the expansion, we obtain a non-perturbative vacuum at zero temperature, which is characterized by

$$\frac{1}{A^2} \sum_{I=1}^D \langle \text{tr}_A [Y^I Y^I] \rangle_{T=0} = \frac{D}{2} \tilde{\lambda}_A^{-1/3}. \quad (7)$$

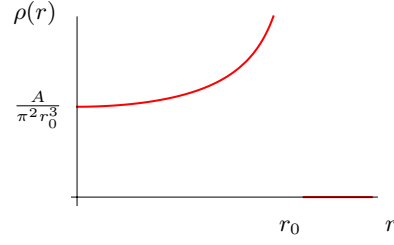


FIG. 2: The nuclear density distribution  $\rho(r)$  obtained in the large  $D$  limit, (10). At the core of the nucleus  $r \sim 0$ , the distribution is homogeneous. For  $r > r_0$ , the density is exactly zero, meaning the complete finiteness of nuclei. A graphical image is drawn in the next figure.

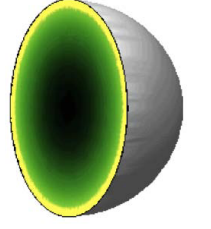


FIG. 3: The density distribution of the nucleus. Darker color means smaller density. Ignoring the surface defect, we find a homogeneous density distribution.

This indeed is consistent with (5). Around this vacuum,  $Y^I$  behaves as a free massive field with a mass  $\tilde{\lambda}_A^{1/3}$ , as the interaction is suppressed by  $1/D$  [33]. By using the propagator of the free massive scalar at zero temperature,

$$\langle Y_{ab}^I(t) Y_{cd}^J(0) \rangle_{T=0} \sim \frac{1}{2\tilde{\lambda}_A^{1/3}} \delta_{ad} \delta_{bc} \delta^{IJ} \quad (t \sim 0), \quad (8)$$

we can evaluate (6) as

$$\begin{aligned} \langle \text{tr}_A \exp(ik \cdot Y) \rangle_{T=0} &= \sum_{n=0}^{\infty} \frac{A}{n!(n+1)!} \left( \frac{-Ak^2}{2\tilde{\lambda}_A^{1/3}} \right)^n \\ &= A \frac{2}{r_0 |k|} J_1(r_0 |k|) \end{aligned} \quad (9)$$

where  $r_0 \equiv (2A/\tilde{\lambda}_A^{1/3})^{1/2}$ . Here we have used a fact that only ladder diagrams [24] contribute to (6). The distribution in our 3-dimensional space can be obtained by a Fourier transform followed by an integration over the holographic directions,

$$\begin{aligned} \rho(r) &= \int \prod_{i=4}^D dx^i \frac{1}{(2\pi)^D} \int d^D k e^{-ik \cdot x} \langle \text{tr}_A \exp(ik \cdot Y) \rangle_{T=0} \\ &= \begin{cases} \frac{A}{\pi^2 r_0^2 \sqrt{r_0^2 - r^2}} & (r < r_0) \\ 0 & (r_0 < r) \end{cases} \end{aligned} \quad (10)$$

where  $r$  is a radial coordinate in the 3-dimensional space spanned by  $\{Y^1, Y^2, Y^3\}$ . *We found that nuclear density is zero outside a certain radius, which is the finiteness of nuclei.*

Plotting this function shows the density distribution given in Fig. 2 and Fig. 3. We notice that at  $r = r_0$  the density goes up, which is not really the case for realistic nuclei. However, since this surface part is at a sub-leading in the large  $A$  expansion, explicit computations of  $1/A$  corrections may be necessary to see the details. Other possible origin may be the formula (6) itself [25]. We hope to come back to this issue in a near future.

We also notice that the isospin structure of the nucleus is invisible, since it is encoded in  $w$ , which is subdominant in the large  $A$  expansion. In contrast, the distribution in  $X^i$  directions indicates that the nucleus includes components of excited baryon resonances.

**Evaluation of nuclear size.** — Finally we shall study numerics. Using the explicit matrix quantum mechanics (2) of [1], we can calculate (4) through (8) as

$$\sqrt{r_{\text{mean}}^2}\Big|_{T=0} = \frac{3^{5/2}\pi^{2/3}}{2^{5/6}5^{1/6}} \frac{A^{1/3}}{M_{\text{KK}}(N_c\lambda^2)^{1/3}}. \quad (11)$$

A natural choice of the experimental inputs should be  $g_{\pi NN} \sim 13.2$  and  $m_\rho \sim 776$  [MeV] as these dictate nuclear forces (the model [1] is in the chiral limit). From the work [13] we find that these experimental inputs are reproduced when  $\lambda = 5.31$  and  $M_{\text{KK}} = 949$  [MeV]. This input leads us to a nuclear mean square radius

$$\sqrt{r_{\text{mean}}^2} \sim 0.7A^{1/3} [\text{fm}]. \quad (12)$$

As an order estimate, this is quite close to the observed value  $\sqrt{r_{\text{mean}}^2} \simeq 1.0A^{1/3}$  [fm] (which is equivalent to the well-known nuclear radius  $R \simeq 1.2A^{1/3}$  [fm] as  $r_{\text{mean}}^2 = (3/5)R^2$  for a homogeneous nuclear density.)

One can alternatively think of  $r_0$  in the density distribution (10) as the nuclear radius. After going back from  $Y$  to  $X$ , with the same experimental values as the inputs, we obtain  $r_0 \sim 0.8 \times A^{1/3}$  [fm]. This again is consistent with experiments.

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- [26] This was first described in [5], and a precise matrix action in the realistic holographic QCD model was derived in [1].
- [27] Most of known examples are in this category. The D3-D(-1) background [7] is an exception where distributed D(-1)-branes play the role of the IR end, resulting in a similar effect on baryon vertices.
- [28] In the Sakai-Sugimoto model, the baryon vertex is absorbed into the flavor D-brane [10] and can be thought of as a soliton solution on the flavor D-brane effective field theory [11, 12]. The soliton is a generalization of the Skyrmion, and static properties of baryons can be analyzed [13, 14] (see also [15, 16]).
- [29] This is standard for any brane model of holographic QCD, though no consistent truncation has been known.
- [30] A 1-dimensional Chern-Simons term  $\int dt \text{tr} A_t$  in the action contributes only the overall  $U(1)$  sector of the  $U(A)$ , thus is a sub-leading order in the large  $A$  expansion.
- [31] The field  $X^5$  was not written in [1], but together with  $X^4$  they span the transverse cigar geometry.
- [32] The contribution of the  $Y^I$  is dominant at the zero temperature. However, since the model (3) would be in a confinement phase in low temperature [22], the thermal excitation of  $Y^I$  is suppressed. Thus the  $1/A$  corrections of the model would be relevant in a finite temperature.
- [33] If the dynamical mass  $\tilde{\lambda}_A^{1/3}$  is larger than  $m_Y$ , we can ignore the effects of  $m_Y$  in (3). This is the case in our model (2). On the other hand, if  $m_Y$  is large enough, one can integrate out  $X^i$  and again the effects can be ignored by effectively treating  $D = 3$ .